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# The transition from regular to Mach reflexion and from Mach to regular reflexion in truly non-stationary flows

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An experimental investigation on the IHSM 4 × 8 cm Shock Tube has confirmed the hypothesis of Ben-Dor (1978) and Ben-Dor & Glass (1979) that in truly non-stationary flows the transitions from regular to Mach reflexion (RR → MR) and from Mach to regular reflexion (MR → RR) are different. Consequently it is shown that a hysteresis loop exists in the RR ⇄ MR transition phenomenon.

## 1. Introduction

The reflexion of oblique shock waves is a nonlinear problem which has been investigated analytically and experimentally by many researchers. Four different shock-wave reflexions have been observed in pseudo-steady flows in shock tubes over two-dimensional straight wedges. They are (figure 1); (*a*) regular reflexion (RR); (*b*) single-Mach reflexion (SMR); (*c*) complex-Mach reflexion (CMR); and (*d*) double-Mach reflexion (DMR). Owing to the geometry of the models used to reflect oblique shock waves in a steady flow, CMR and DMR cannot materialize in steady flows, even though the flow behind the reflected shock wave can be supersonic (Ben-Dor 1978, 1980), and hence, only RR (figure 2*a*) and SMR (figure 2*b*) are possible.

Although RR and SMR were discovered more than 100 years ago (Mach 1878), almost no work was done on this problem until von Neumann (1963) re-initiated the problem in the early 40's. The intensive investigation that followed von Neumann's work finally led to the discovery of CMR by Smith (1945) and DMR by White (1951). Since then investigators were mostly interested in establishing the correct transition criterion between the various reflexions.

Naturally, the first transition to be investigated was the RR ⇄ SMR transition. Since it was found throughout the past years that the termination of RR can result in, in pseudo-stationary flows, SMR, CMR or DMR, the notation RR ⇄ MR (MR indicates Mach reflexion) will be used in the following. A detailed discussion on whether SMR, CMR or DMR form when RR terminates is given by Ben-Dor & Glass (1979). Until 1975, two different criteria existed for the transition RR ⇄ MR. This transition was regarded as equivalent for both steady and non-stationary flows which could be made pseudo-steady by attaching the frame of reference to the incident shock wave.

These two criteria (see subsequent discussion) are known as the 'detachment' criterion of von Neumann (1963) and the 'mechanical-equilibrium' criterion of

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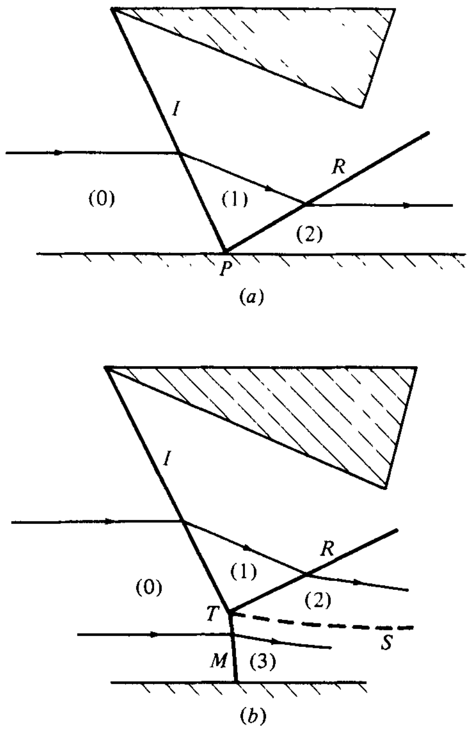
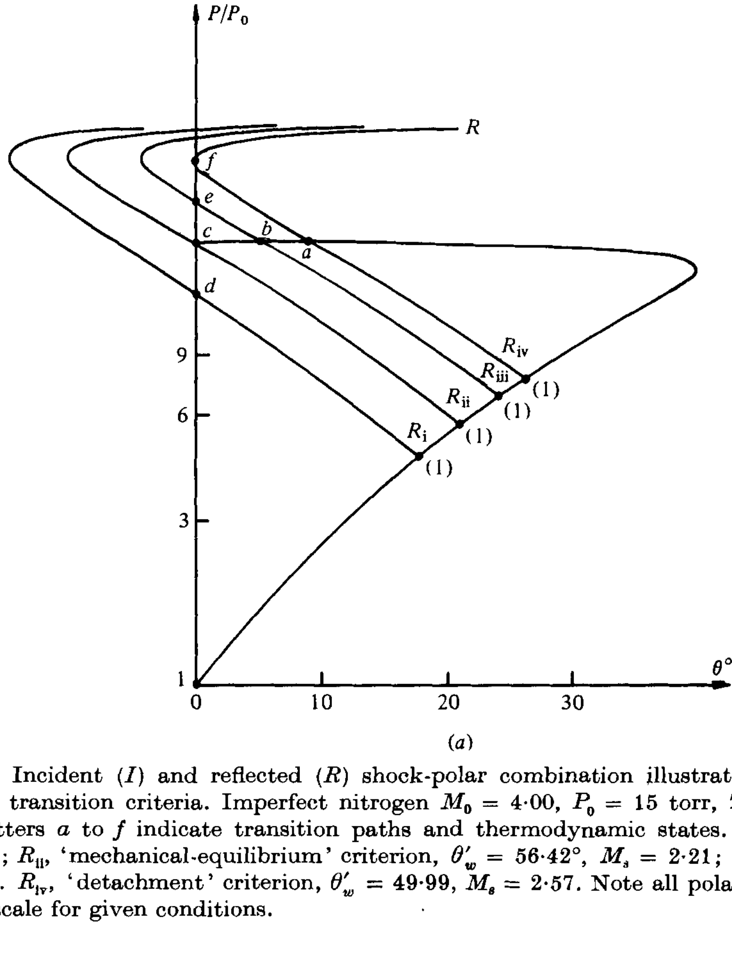



FIGURE 2. Illustration of possible oblique shock wave reflexions in steady flows: (a) regular reflexion (RR); (b) single-Mach reflexion (SMR).

Henderson & Lozzi (1975). Recently, Hornung, Oertel & Sandeman (1979) forwarded a new criterion, the 'length scale' criterion, that resulted in different transition lines for steady and pseudo-steady flows. In a detailed investigation of the reflexion phenomena in non-stationary (pseudo-steady) flows, Ben-Dor (1978) and Ben-Dor & Glass (1979) forwarded a hypothesis concerning the RR $\rightleftarrows$ MR transition in truly non-stationary flows. By truly non-stationary flows, we mean flows which cannot be made pseudo-steady. They hypothesized (see subsequent discussion) that in truly non-stationary flows the RR $\rightarrow$ MR transition need not be the same as the MR $\rightarrow$ RR transition. The main assumption behind the hypothesis is that once RR or MR is formed, and then $\theta_w$ or $M_s$ is changed gradually to cause transition to MR or RR, the original reflexion (RR or MR) will terminate only when physically it becomes impossible for it to exist.

Using streak camera technique with curved slits, we were able to verify the hypothesis of Ben-Dor (1978) and Ben-Dor & Glass (1979). It was found that in truly non-stationary flows the RR $\rightarrow$ MR transition is indeed different from the MR $\rightarrow$ RR transition criterion. For example, at $M_s = 4$ the RR $\rightarrow$ MR transition occurs at a wedge angle $\theta_w \approx 40°$ while the MR $\rightarrow$ RR transition takes place at $\theta_w \approx 65°$.




(*a*)

FIGURE 3. Incident (*I*) and reflected (*R*) shock-polar combination illustrating the different RR ⇄ MR transition criteria. Imperfect nitrogen $M_0 = 4{\cdot}00$, $P_0 = 15$ torr, $T_0 = 300$ K. For clarity, letters *a* to *f* indicate transition paths and thermodynamic states. $R_{\rm i}$, $\theta'_w = 60{\cdot}00°$, $M_s = 2{\cdot}00$; $R_{\rm ii}$, 'mechanical-equilibrium' criterion, $\theta'_w = 56{\cdot}42°$, $M_s = 2{\cdot}21$; $R_{\rm iii}$, $\theta'_w = 52{\cdot}66°$, $M_s = 2{\cdot}43$. $R_{\rm iv}$, 'detachment' criterion, $\theta'_w = 49{\cdot}99$, $M_s = 2{\cdot}57$. Note all polars are accurately drawn to scale for given conditions.

## 2. Present status of the RR ⇄ MR transition criterion in steady and pseudo-steady flows

Three different criteria for the transition RR ⇄ MR exist in the literature. The most quoted criterion is due to von Neumann (1963), and is based on the fact that in RR the deflexion of the flow, $\theta_2$, by the reflected shock wave R is equal in magnitude but opposite in sign to the deflexion $\theta_1$ by the incident shock wave *I*. Therefore, $\theta_1 + \theta_2 = 0$. This is violated when $\theta_1$ exceeds in magnitude the maximum deflexion angle $\theta_{2m}$. This criterion that is referred to as the detachment criterion (the term detachment comes from steady flows where the oblique shock-wave detaches at this angle) has the following analytical form:

$$\theta_1 + \theta_{2m} = 0. \tag{1}$$

The detachment criterion can be illustrated best by using the pressure-deflexion $(P, \theta)$-shock polars. Consider figure 3, where the *I* and *R* polars represent the incident and reflected shock waves, respectively. Since the net deflexion through a RR is zero state (2) behind *R* (figures 1*a* and 2*a*) is at the point where the *R* polar intersects the

$P/P_0$ axis, e.g., point $d$ on $R_{\text{i}}$ (figure 3). As the angle $\theta_1$ increases the $R$ polar moves away from the $P/P_0$ axis until it becomes tangent to it (point $f$ on $R_{\text{iv}}$). Upon a further increase in $\theta_1$, the $R$ polar does not intersect the $P/P_0$ axis anymore and a RR is not possible. Consequently, the detachment criterion is illustrated by the $R_{\text{iv}}$ polar.

Due to disagreement between the experiments of Smith (1945) and others and the detachment criterion at very weak incident shock waves, alternative criteria were sought by various investigators. Henderson & Lozzi (1975) introduced an alternative criterion which has the property that the system always remains in mechanical equilibrium (i.e., no pressure discontinuities). Consider figure 3 and note that once RR terminates and MR forms, the solution moves from a RR at the point where the $R$ polar is tangent to the $P/P_0$ axis (point $f$ on $R_{\text{iv}}$) to a MR at the point where the $I$ and the $R$ polars intersect (point $a$ on $R_{\text{iv}}$). Consequently, a pressure change (from $P_f$ to $P_a$), is associated with the transition, if the detachment criterion is accepted. Henderson & Lozzi (1975) argue that 'a system which develops a pressure discontinuity during transition cannot be in mechanical equilibrium'. Furthermore, they say that 'if a discontinuity occurs during transition than an unsteady wave of finite amplitude or a finite amplitude band of waves will be generated in the flow. These would be expansion [waves] for RR → MR and compression [waves] for MR → RR.' Since these waves have never been observed experimentally and since the detachment criterion failed to agree with their experiments they abandoned the detachment criterion and suggested an alternative. Their alternative criterion, the 'mechanical-equilibrium' criterion enables the system to remain in mechanical equilibrium during transition. They argued that in order to maintain the system in mechanical equilibrium, the RR ⇄ MR transition should take place at the point where the $R$ polar intersects the $I$ polar (MR solution) at the same point where it intersects the $P/P_0$ axis (RR solution) as illustrated by point $c$ on the $R_{\text{ii}}$ polar of figure 3. Consequently, the mechanical-equilibrium criterion implies that

$$\theta_1 + \theta_2 = \theta_3 = 0, \tag{2}$$

where $\theta_3$ is the deflexion of the flow while passing through the Mach stem. Consider now polar $R_{\text{iii}}$ (figure 3) and note that according to the mechanical-equilibrium criterion, a MR will take place at point $b$, since the transition criterion described by $R_{\text{ii}}$ has been exceeded. However, according to the detachment criterion a RR will occur at point $e$ since the transition criterion described by $R_{\text{iv}}$ has not been reached. For all the polar combination between $R_{\text{ii}}$ and $R_{\text{iv}}$, RR and MR are theoretically possible. Figure 4 illustrates the size of the dual-solution region in the $M_s$, $\theta'_w$ plane ($\theta'_w = \theta_w + \chi$, where $\chi$ is the triple-point trajectory angle). It is seen that the area of disagreement between the mechanical equilibrium and detachment criteria is very large.

Although Henderson & Lozzi (1975) reported that excellent agreement was obtained between the mechanical-equilibrium criterion and their wind-tunnel experiments, there are unfortunately some difficulties concerning it.

First of all, it does not apply over the entire range of $M_0 > 1$ ($M_0 = M_s \operatorname{cosec} \theta'_w$). The condition given by equation (2) is not always possible. Kawamura & Saito (1956) showed that for flow Mach number $M_0$ smaller than a certain 'change-over' value $M_{0c}$, the $R$ polar becomes tangent to the $P/P_0$ axis inside the $I$ polar, and the situation

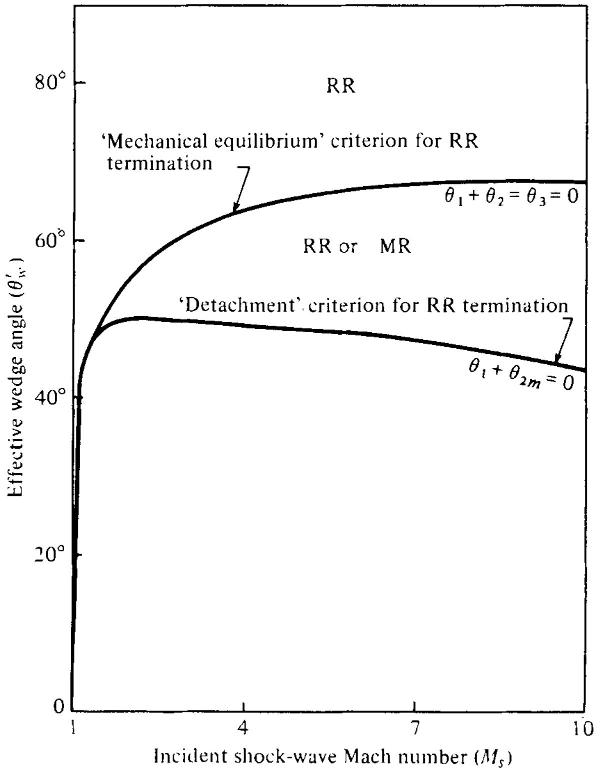



FIGURE 4. Comparison between the 'detachment' and the 'mechanical-equilibrium' criteria for RR termination in the $M_s$, $\theta'_w$ plane, imperfect nitrogen $P_0 = 15$ torr, $T_0 = 300$ K.

described by (2) is unobtainable. The precise value of $M_{0c}$ is still not clear since different values are reported by different investigators (see Ben-Dor & Glass 1979 for details). For a diatomic gas $M_{0c} = 2{\cdot}20 \pm 0{\cdot}03$ while for a monatomic gas $M_{0c} = 2{\cdot}46 \pm 0{\cdot}01$. Consequently, the mechanical-equilibrium criterion exists only for $M_0 \geqslant M_{0c}$. Unfortunately, the detachment criterion which does exist in the range $1 < M_0 \leqslant M_{0c}$ disagrees completely with the experiments of Smith (1945), Bleakney & Taub (1949), White (1951), Kawamura & Saito (1956), Henderson & Lozzi (1975) and others. Consequently, the RR $\rightleftarrows$ MR transition criterion in the range $1 < M_0 \leqslant M_{0c}$ is unknown, and hence, yet to be found.

Second, the attempt of Henderson & Lozzi (1975) to substantiate their idea in pseudo-steady (shock-tube) flows revealed, as they say, a 'remarkable anomaly' between their results from wind-tunnel and shock-tube experiments with single, two-dimensional straight wedges, where RR continued to exist in the shock-tube experiments beyond the perfect-gas limit predicted by both the detachment and the mechanical-equilibrium criteria. Henderson & Lozzi (1975) resolved the anomaly by advancing that the RR configurations observed beyond the limit predicted by their criterion were undeveloped DMR configurations in which all the shock waves, slipstreams and triple-points typical of a well-developed DMR (figure 1*d*) were too close together to be observed. Some criticism concerning this hypothesis was given by Ben-Dor & Glass (1979) and hence its validity needs further proof.

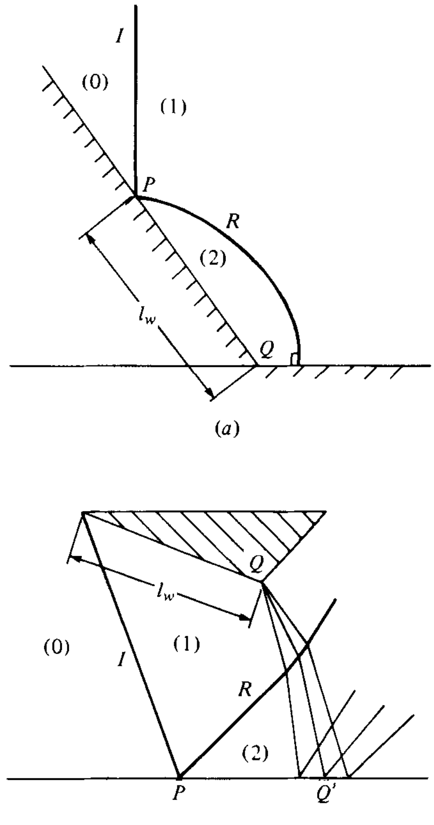



FIGURE 5. Communication of a scale length $l_w$ to the reflexion point $P$: ($a$) non-stationary flow; ($b$) steady flow.

Auld & Bird (1976) who decided to approach the RR $\rightleftarrows$ MR transition problem numerically arrived at a similar conclusion. They studied the transition numerically in steady flows in the region where both RR and SMR are theoretically possible (the region between $R_{\text{ii}}$ and $R_{\text{iv}}$ of figure 3). Their calculation was carried out at the molecular level using a direct simulation Monte-Carlo method. Since a RR was established in the 'dual-solution region' in both monatomic and diatomic steady flows, Auld & Bird (1976) concluded that 'the recent conclusion [of Henderson & Lozzi (1975)] that Mach reflexion [MR] always occurs in the overlap region (figure 4) requires further study'. They suggested furthermore that 'low density wind tunnel results that resolve the wave structure of the reflexion point would be particularly useful'.

Hornung *et al.* (1979) initiated recently another criterion for the termination of RR. They argued that in order for a MR to form, i.e. a curved Mach stem to establish, a length scale must be available at the reflexion point, i.e., pressure signals must be communicated to the reflexion point $P$ of a RR (figures 1$a$ and 2$a$). This single argument eventually led to two different termination lines for RR depending on whether the flow under consideration is steady or pseudo-steady.

Consider the pseudo-steady RR in figure 5($a$) and note that the length $l_w$ can affect the reflexion point $P$ only when a subsonic flow is established between $Q$ and $P$ (in a

| Type of flow ... | Steady flow | | Pseudo-steady flow | |
|---|---|---|---|---|
| Investigators | $M_0 < M_{0c}$ | $M_0 > M_{0c}$ | $M_0 < M_{0c}$ | $M_0 > M_{0c}$ |
| Henderson & Lozzi (1975) | Unknown | $\theta_1+\theta_2 = \theta_3 = 0$ | Unknown | $\theta_1+\theta_2 = \theta_3 = 0$ |
| Hornung *et al.* (1979) | $\theta_1+\theta_{2s} = 0$ | $\theta_1+\theta_2 = \theta_3 = 0$ | $\theta_1+\theta_{2s} = 0$ | $\theta_1+\theta_{2s} = 0$ |

TABLE 1. RR $\rightleftarrows$ MR transition criteria in steady and pseudo-steady flows.

frame of reference attached to $P$). In a steady flow (figure 5$b$) the length $l_w$ can effect the reflexion point $P$ only if a propagation path exists between point $Q$ and point $P$ via the expansion wave at $Q'$. This is possible only if the flow between $P$ and $Q'$ is subsonic. According to Hornung *et al.* (1979) this could happen if a MR existed since the flow behind the Mach stem is subsonic. Consequently, they argue that transition takes place the very first time MR becomes theoretically possible. Consider figure 3 and note that this corresponds to point $c$ on $R_{\text{ii}}$, which also corresponds to the mechanical-equilibrium criterion of Henderson & Lozzi (1975).

Thus, the analysis of Hornung *et al.* led to two different termination lines. In steady flow it predicts transition at the same point where the mechanical-equilibrium criterion of Henderson & Lozzi (1975) does, and in pseudo-steady flows it predicts transition at:

$$\theta_1 + \theta_{2s} = 0. \tag{3}$$

Note that the transition lines described by equation (1) and (3) are almost identical. They are too close together to be resolved experimentally. Consequently, the above-mentioned disagreement between experiments and the detachment criterion at weak incident shock waves, is still not resolved. Even the inclusion of real-gas effect (Ben-Dor 1978; Ben-Dor & Glass 1979) does not account in this range of Mach numbers for the persistence of RR beyond the limit predicted by the perfect gas theory.

The 'length-scale' criterion has different formulations in steady flows. For $M_0 \geqslant M_{0c}$ it is described by (2) and for $M_0 \leqslant M_{0c}$ it reduces to the 'sonic' criterion, i.e. RR $\rightleftarrows$ MR transition occurs when the flow behind the reflected shock wave $R$ becomes subsonic. Analytically, the sonic criterion can assume one of the following forms in the region $M_0 < M_{0c}$:

$$\theta_1 + \theta_{2s} = 0, \quad M_2 = 1. \tag{4$a,b$}$$

In pseudo-steady flows, equations (1) or (4) is the transition criterion for the entire range of $M_0$. For this case the flow behind $R$ (figure 1$a$) should be subsonic with respect to the reflexion point $P$, i.e.

$$M_{2P} = 1. \tag{4$c$}$$

The foregoing discussion concerning the RR $\rightleftarrows$ MR transition criterion is summarized in table 1. It can be seen that while Henderson & Lozzi (1975) regard the transition criteria to be identical for steady and pseudo-steady flows, Hornung *et al.* (1979) suggest different criteria. As a matter of fact Henderson & Lozzi's and Hornung *et al.*'s RR $\rightleftarrows$ MR transition criteria agree only for steady flows in the range $M_0 > M_{0c}$. Recall that the difference between their transition lines in pseudo-steady flows is shown in figure 4.

## 3. The RR $\rightleftarrows$ MR transition in truly non-stationary flows

As mentioned earlier, Ben-Dor (1978) and Ben-Dor & Glass (1979) hypothesized that once a reflexion (RR or MR) is established in a truly non-stationary flow (a flow which cannot be made pseudo-steady) and either $M_s$ or $\theta_w$ are changed continuously to force transition, the original reflexion will terminate only when it becomes impossible for it to exist. Consider figure 3 and note that if one starts with a given MR at point $a$ and the wedge angle is increased slowly, the hypothesis suggests that states $a$ (MR), $b$ (MR), $c$ (MR $\rightarrow$ RR) and $d$ (RR) will be encountered. However, if one starts with a given RR at point $d$, and $\theta_w$ is decreased gradually, the hypothesis suggests that states $d$ (RR), $c$ (RR), $e$ (RR), $f$ (RR) and $a$ (MR) will be encountered. Note that the former sequence of events follows a continuous transition at point $c$ while the latter path undergoes a discontinuous transition from point $f$ to $a$. If proved correct, the hypothesis suggests that a hysteresis exists in the RR $\rightleftarrows$ MR transition process, i.e., if one starts at point $a$ with a MR and the wedge angle is first increased to cause transition to RR and then decreased to force the reflexion back to the original MR, the following states will be encountered: $a$ (MR), $b$ (MR), $c$ (MR $\rightarrow$ RR), $d$ (RR), $c$ (RR), $e$ (RR), $f$ (RR), $a$ (MR). The hysteresis loop is described on figure 3 by states $a$, $c$ and $f$. Consider figure 4 and note that if one starts with a MR below the detachment transition line and $\theta'_w$ (or $\theta_w$) is increased the transition MR $\rightarrow$ RR will occur upon passing through the mechanical-equilibrium transition line. If now $\theta'_w$ is decreased the RR will terminate at the detachment criterion, and MR will form.

In view of the foregoing discussion it seems appropriate to call this criterion, (if proved correct) the 'inertia' criterion.

## 4. Experiments

The discussion clearly indicated that in order to verify the 'inertia' hypothesis, one has to perform an experiment in which either $M_s$ or $\theta_w$ (or both) is changed continuously, in order to force the initial reflexion RR or MR to undergo transition to MR or RR, respectively. Since changing $\theta_w$ seems more practical and promising it was decided to reflect a constant-velocity shock wave over curved wedges.

The experiments were conducted on the IHSM $4 \times 8$ cm Pressure-Driven Shock Tube (Takayama, Honda & Onodera 1977). Three types of curved wedges (figure 6) were used. In the concave cylinder (model $A$, figure 6$a$), $\theta_w$ increases continuously from 0 to 90°. In the convex cylinders (models $B$ and $C$, figures 6$b$ and 6$c$, respectively), $\theta_w$ decreases continuously from 90° to 0 in model $B$, and from 53·13° to 0 in model $C$. Consequently, while the transition MR $\rightarrow$ RR takes place in model $A$, the RR $\rightarrow$ MR transition occurs in models $B$ and $C$.

The incident shock wave Mach number range was $1{\cdot}1 \leqslant M_s \leqslant 4{\cdot}0$. The shock wave velocity was measured with pressure transducers (Kistler model 606) and a digital counter. The pressure transducers were located 12 cm apart, just ahead of the test section. The effects of the attenuation and acceleration of the shock wave were checked in the preparatory experiments. They were found to be negligible.

Dry air at initial pressures of $3 < P_0 < 150$ torr was used as test gas. Ben-Dor (1978) and Ben-Dor & Glass (1979) have shown recently that in the range $M_s < 6$ the initial pressure does not influence the RR $\rightleftarrows$ MR transition line. Consequently, the initial

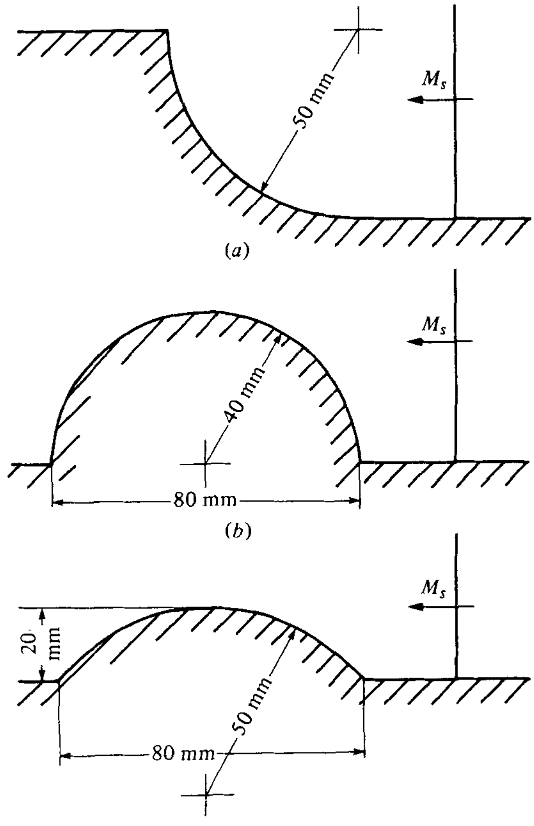



FIGURE 6. Scale drawings of the three different models used in the present study: (a) model $A$, concave; (b) model $B$, convex; (c) model $C$, convex.

pressure could be considered practically constant. The initial temperature $T_0$ throughout all the experiments was about 300 K.

The non-stationary phenomenon was recorded using shadowgraph and schlieren methods. A giant-pulse ruby laser with a 20 ns light-pulse width was used to obtain the instantaneous (single shot) photographs. For the continuous observation of the shock diffraction process an Imacon high-speed camera (John Hadland Model 700) was used, in both the framing and the streak modes. Many more details concerning the experimental set-up and technique were reported by Takayama & Kawauchi (1979).

In previous experimental investigations, the value of $\theta_w$ at which transition occurred i.e., $\theta_{w\text{tr}}$ was deduced by interpolating between many single-shot photographs, which were obtained by either changing $\theta_w$ for a fixed $M_s$, or changing $M_s$ for a constant $\theta_w$. However, this kind of experiment always involves a small scatter in the shock-wave Mach number and the induced flow field behind it. Consequently, the determined value of $\theta_{w\text{tr}}$, inevitably involved some experimental errors.

Bazhenova *et al.* (1968) applied the streak camera technique to study the oblique shock-wave diffraction over two-dimensional straight wedges. In their study the shape of the DMR was measured by using a straight slit, which was placed parallel to the direction of propagation of the incident shock wave.

In the present experiments, curved slits, 0·6 and 0·7 mm wide were accurately placed on the test-section windows in a direction that coincides with the curved wedge surface. In this arrangement the exact point of transition along the wedge surface could be determined for each experiment.

It is worth noting, that in order to increase the resolution (accuracy) in measuring the transition point (wedge angle) along the concave-curved wedge (model $A$, figure 6$a$), the image of the curved slit was rotated by 38° using an image rotator in front of the objective lens of the Imacon high speed camera.

## 5. Results

Typical instantaneous shadowgraphs of MR and RR over the concave (model $A$) and the convex (model $B$) wedges are shown in figures 7 and 8, respectively. The incident shock wave is moving from right to left. For model $A$ (figure 7), the resulted reflexion is initially a MR (figure 7$a$). As the incident shock wave proceeds along the wedge, the reflexion undergoes transition to RR (figure 7$b$). In the case of model $B$ (figure 8), the opposite sequence of events is obtained. First a RR (figure 8$a$) is formed, then the transition RR$\rightarrow$MR takes place to result in a MR (figure 8$b$). Note that the wave interactions behind the RR shown in figure 7($b$) are no longer similar to that shown in figure 8($a$).

Typical streak photographs as well as explanatory sketches of the shock wave diffraction over models $A$ and $B$ are shown in figures 9($a$) and 9($b$), respectively.

The transition MR$\rightarrow$RR is recorded on the streak photograph shown in figure 9($a$). The length of the observation field, from the end of the concave wedge (point $E$, figure 9$a$) is 74·6 mm. The sweeping speed of the streak is $\frac{2}{3}$ mm $\mu s^{-1}$. Since an image rotator was used with this model (see earlier remarks), the image of the curved 0·6 mm wide slit is inclined by 38° from its normal position, and the incident shock wave is seen to be moving from left to right, in the streak photograph. The two trajectories which are seen at the beginning of figure 9($a$) correspond to the Mach stem $M$ and the slipstream $S$ (figure 7$a$). The transition MR$\rightarrow$RR occurs at point $T$ where a transmitted shock $I$ (incident shock wave of RR) and its reflected shock wave $R$ appear. A clear change in the gradients of the trajectories of $M$ and $I$ is seen at the transition point $T$. A short time after transition takes place, some new trajectories appear in the streak photograph. These trajectories correspond to the previously mentioned shock-wave interactions behind the RR (figure 7$b$).

The transition RR$\rightarrow$MR is recorded on the streak photograph shown in figure 9($b$). The length of the observation field, from the beginning of the convex wedge (point $O$, figure 9$b$) is 75·9 mm. The sweeping speed of the streak is again $\frac{2}{3}$ mm $\mu s^{-1}$. Since image rotators were not used for this model, the incident shock wave is moving from right to left. The two trajectories which are seen at the beginning of figure 9($b$) correspond to the incident ($I$) and reflected ($R$) shock waves of the RR (figure 8$a$). While the trajectory of $I$ is white, the trajectory of $R$ is black. Despite the constant incident shock wave velocity, the trajectory of $I$ looks as if the incident shock wave has accelerated at the starting point $O$. This is due to the finite width (0·7 mm) of the slit. This finite width of the slit is also the reason for the broadening of the trajectory of $R$ near the starting point $O$. As the reflection moves along the wedge the trajectories of $I$ and $R$ approach each other asymptotically, until they converge at the transition point $T$.

At this point, two new trajectories appear. One corresponds to the Mach stem $M$ and the other to the slipstream $S$ of the MR (figure 8*b*). Since the velocity of $I$ is different from that of $M$ the gradients of their trajectories are discontinuous at the transition point $T$.

The foregoing description of the streak photographs indicates how powerful the present method is in precisely determining the point of transition (i.e., the value of $\theta_{wtr}$ from RR to MR and from MR to RR. The value of $\theta_{wtr}$ is obtained from:

$$\theta_{wtr} = \sin^{-1}\frac{l}{R} + \theta_R \tag{5}$$

where $\theta_R$ is the angle by which the image of the slit is rotated with respect to the orientation of the slit ($\theta_R = 38°$ for model $A$ and $0°$ for models $B$ and $C$), $R$ is the radius of curvature of the surface of the model ($R = 50$ mm for model $A$ and $C$ and 40 mm for model $B$) and $l$ is the horizontal distance from the centre of the curvature of the cylinder surface $C$ to the transition point $T$ (figure 9).

Using the elementary-error estimation method, equation (5) results in:

$$\Delta(\theta_{wtr}) = \frac{1}{\cos(\theta_{wtr} - \theta_R)} \cdot \frac{l\Delta(R) + R\Delta(l)}{R^2} + \Delta(\theta_R) \tag{6}$$

where $\Delta(\phi)$ is the absolute error associated with the measurement of the quantity '$\phi$'. Since $l$ and $R$ are of the same order of magnitude and $\Delta(R) \ll \Delta(l)$, equation (6) can be reduced for practical purposes to:

$$\Delta(\theta_{wtr}) = \frac{1}{\cos(\theta_{wtr} - \theta_R)} \frac{\Delta(l)}{R} + \Delta(\theta_R). \tag{7}$$

Finally, if $\Delta(l) = 0{\cdot}2$ mm and $\Delta(\theta_R) = 1°$, the maximum possible errors in calculating the transition angle $\theta_{wtr}$ from equation (6) are $1{\cdot}3°$ for model $A$ and $0{\cdot}3°$ for models $B$ and $C$ for which $\theta_R = 0$.

The actual transition angle $\theta_{wtr}$ as measured in the present experiments, as a function of inverse pressure ratio across the incident shock wave $\xi$, for models $A$ (squares), $B$ and $C$ (filled and open circles, respectively) are shown in figure 10. The transition lines corresponding to equations (2) [line $A$] and (1) [line $B$] are also drawn in figure 10. These lines are calculated for straight wedges. The solid lines are for a perfect diatomic gas ($\gamma = \frac{7}{5}$) and the dashed lines are for imperfect nitrogen in vibrational equilibrium ($T_0 = 300$ K). It is seen that real gas effects become noticeable at $M_s \approx 1{\cdot}5$ [$M_s^2 = (\gamma-1)/2\gamma + (\gamma+1)/(2\gamma\xi)$]. Curves $C$ and $D$ are the RR termination criterion as obtained by Heilig (1969) and Itoh & Itaya (1978) in their study of the RR → MR transition over convex wedges, using Whitham's ray-shock theory.

The experimental results clearly indicate that the RR → MR and the MR → RR transition are different. The higher the incident shock wave Mach number the greater the difference between these two transitions. At $M_s = 4$ the difference in the transition angle $\theta_{wtr}$ is as much as 25° ! The experiments also indicate that $\theta_{wtr}$ does not depend on the curvature of the wedge surface, i.e. on $d\theta_w/dx$ ($x$ is the distance along the shock tube). (Compare experiments obtained by models $B$ and $C$.) The agreement between the experiments and curve $A$ above which a steady or pseudo-steady MR cannot occur and curve $B$ below which a steady or pseudo-steady RR is theoretically impossible, is poor. This we believe is due to the fact that curves $A$ and $B$ are for straight wedges

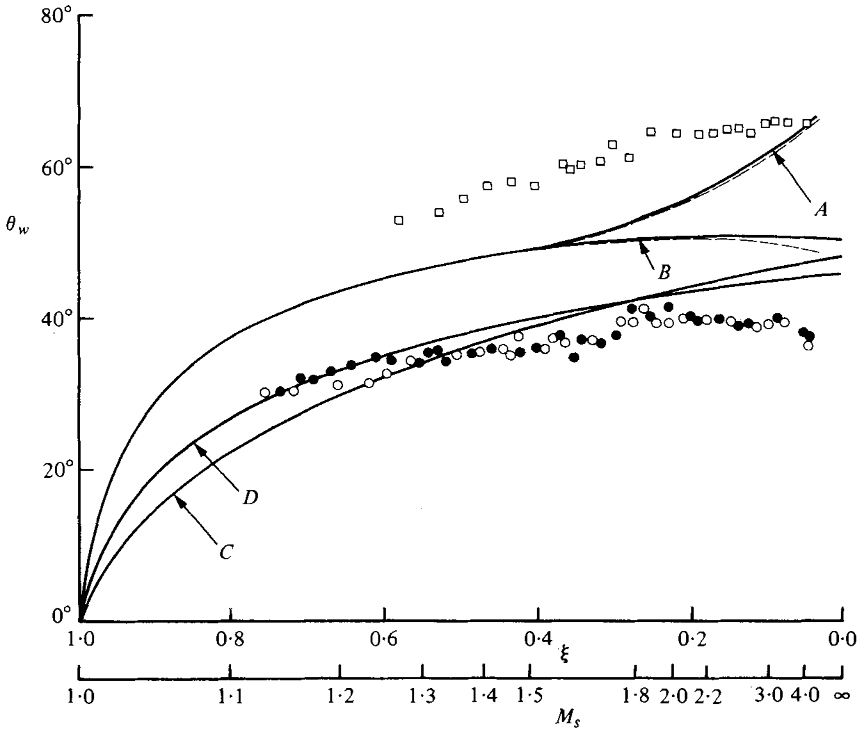



FIGURE 10. Present experimental results of actual transition from MR → RR and from RR → MR in the $\xi$, $\theta_w$ plane and some theoretical transition lines: $\xi$ is the inverse pressure ratio across the incident shock wave; $\theta_w$ is the wedge angle. Symbols: □, experiments over model $A$; ○, experiments over model $B$; ●, experiments over model $C$. Solid lines, perfect diatomic gas ($\gamma = \frac{7}{5}$); dashed lines, imperfect nitrogen in vibrational equilibrium. Line $A$, transition according to equation (2); line $B$, transition according to equation (1); line $C$, Heilig's (1968) analysis for cylindrical wedges; line $D$, Ito & Itaya's (1978) analysis for cylindrical wedges.

whereas our experiments are for curved wedges. Nevertheless, the present experiments prove that in truly non-stationary flows the RR→MR and the MR→RR transitions are significantly different.

As mentioned earlier, curve $C$ shows Heilig's (1969) analytical prediction of the termination of RR over convex wedges. Heilig's analytical prediction is in much better agreement with our RR→MR experiments, than with the von Neumann (curve $B$) prediction. Nevertheless, his predictions are only in fair agreement with our experiment which is estimated to have a maximum error of 0·3°.

A modification of Heilig's analysis was recently published by Itoh & Itaya (1978). Their prediction (curve $D$) is in a better agreement with the present experiment, than that of Heilig. In the range $M_s < 1{\cdot}35$ ($\xi > 0{\cdot}6$) the agreement between curve $D$ and our experiment is very good. For higher Mach numbers the agreement becomes progressively worse. In this range the actual $\theta_{w\mathrm{tr}}$ is smaller than that predicted by Itoh & Itaya. It is worthwhile mentioning that Itoh & Itaya's theory could be improved by including real gas effects which would shift their transition line (curve $D$) further down.

## 6. Discussion

The present experimental results verify the hypothesis of Ben-Dor (1978) and Ben-Dor & Glass (1979) that in truly non-stationary flows the transition RR $\rightarrow$ MR is indeed different from the transition MR $\rightarrow$ RR.

In view of this important finding it is of interest to examine the theoretical basis of the presently existing criteria (for the RR $\rightleftarrows$ MR transition) in steady or pseudo-steady flow, as whether or not they can explain the present results in non-stationary flows.

The actual MR $\rightarrow$ RR transition occurs beyond the limit predicted by the detachment criterion as modified by Heilig (1969) and Itoh & Itaya (1978) for non-stationary flows. Consequently, the detachment criterion cannot explain the present experiments.

The present results for the RR $\rightarrow$ MR transition, indicate that it follows the detachment criterion as modified by Heilig (1969) and Itoh & Itaya (1978) for convex wedges. Therefore, it does not agree with the mechanical equilibrium concept which is based on the idea that pressure discontinuities cannot exist during transition unless a compression wave is generated. Although the present experiments did not reveal any compression wave there is a possibility that the resolution of the present experiments was not good enough to indicate clearly whether or not a compression wave exists during transition. Consequently, although there is evidence that the present experiments contradict the mechanical equilibrium concept, this concept cannot be discarded.

The length scale concept of Hornung *et al.* (1979) is contradicted by our RR $\rightarrow$ MR experiments. Their criterion implies that the transition should take place the first time a length scale is available at the reflexion point. However, owing to our use of curved wedges, a length scale (radius of curvature) is available at the reflexion point any time. Consequently, the RR $\rightarrow$ MR transition should have followed line $A$ (figure 10) rather than line $D$.

The foregoing discussion indicates the significance of our findings. It suggests that the presently existing concepts for the RR $\leftrightarrows$ MR transition, i.e., detachment, mechanical equilibrium and length scale, cannot explain the phenomenon in truly non-stationary flows.

It is our belief that a general criterion (concept) which will actually predict the RR $\rightleftarrows$ MR transition in steady, pseudo-steady and non-stationary flows should exist. Consequently, upon resolving the previously mentioned disagreement between Henderson & Lozzi (1975) and Hornung *et al.* (1979) [see table 1] this general criterion should reduce in (figure 10):

(1) steady flows, to line $A$;

(2) pseudo-steady flows, to line $A$ if Henderson & Lozzi are correct, or line $B$ if Hornung *et al.* are correct;

(3) non-stationary flows, to line $D$ for convex wedges and a line which will agree with the present results for concave wedges.

## 7. Conclusion

An experimental investigation on the IHSM 4 × 8 cm shock tube of the RR ⇄ MR transition in truly non-stationary flows, revealed that the transitions RR → MR and MR → RR are different.

While verifying the hypothesis of Ben-Dor (1978) and Ben-Dor & Glass (1979), the results suggest that the presently existing concepts for the RR ⇄ MR transition cannot serve as general criteria, since they cannot explain the phenomenon in non-stationary flows. Consequently, the question 'What is the general criterion (concept) that explains the RR ⇄ MR transition in steady, pseudo-steady and non-stationary flows?' is reopened.

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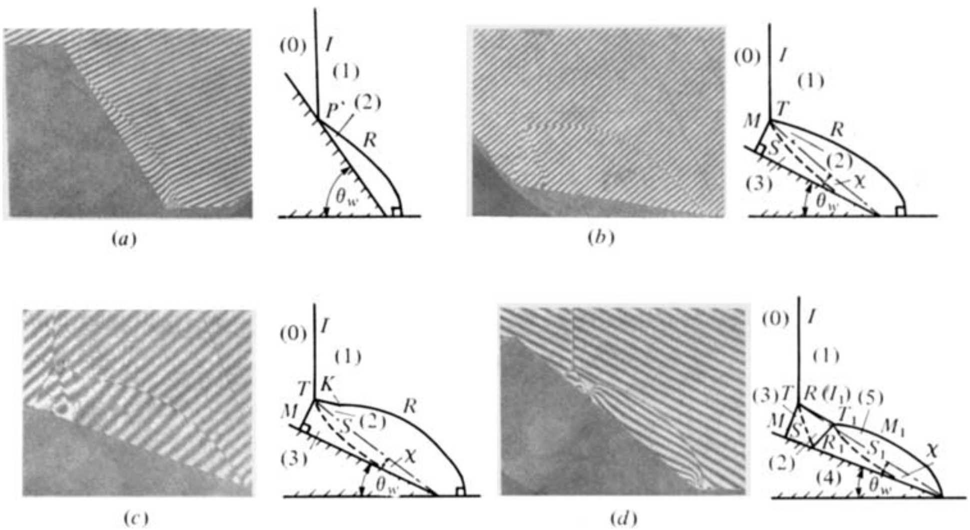
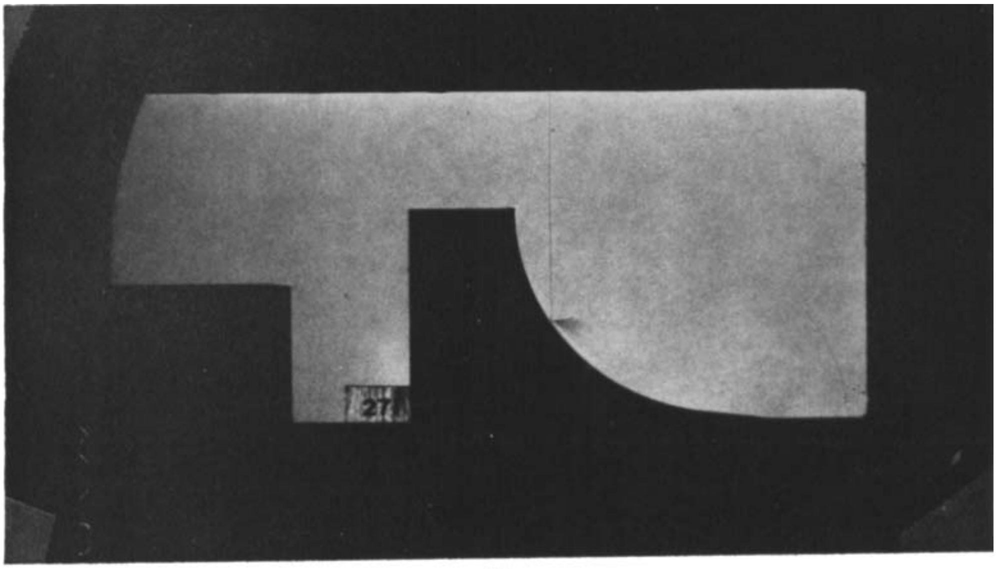
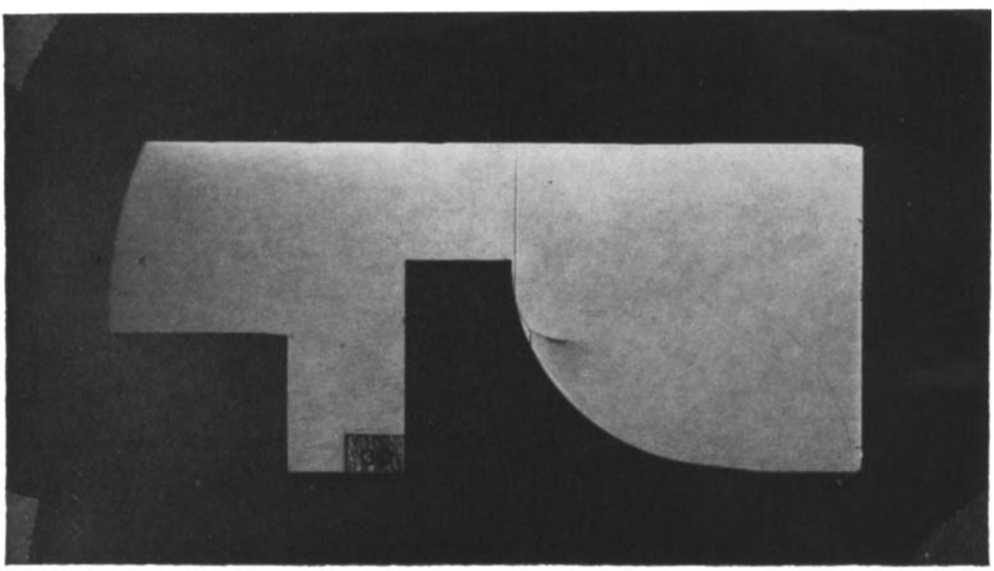



FIGURE 1. Illustration of four possible oblique shock-wave reflexions. (Interferograms are on the left and explanatory sketches on the right.) The interferograms ($\lambda = 6943$ Å) were taken with a 23 cm diameter Mach–Zehnder interferometer in the UTIAS $10 \times 18$ cm Hypervelocity Shock Tube for nitrogen at an initial pressure $P_0 \approx 15$ torr and temperature $T_0 \approx 300$ K. $I$, $I_1$, incident shock waves: $R$, $R_1$, reflected shock waves; $M$, $M_1$, Mach stems; $S$, $S_1$, slipstreams; $T$, $T_1$, triple points; $\chi$, $\chi'$, triple point trajectory angles; (0) to (5), thermodynamic states. (*a*) Regular reflexion (RR), wedge angle $\theta_w = 60°$, shock Mach number $M_s = 4{\cdot}68$. (*b*) Single-Mach reflexion (SMR), $\theta_w = 10°$, $M_s = 4{\cdot}72$. (*c*) Complex-Mach reflexion (CMR), $\theta_w = 20°$, $M_s = 6{\cdot}90$. (*d*) Double-Mach reflexion (DMR), $\theta_w = 40°$, $M_s = 3{\cdot}76$.

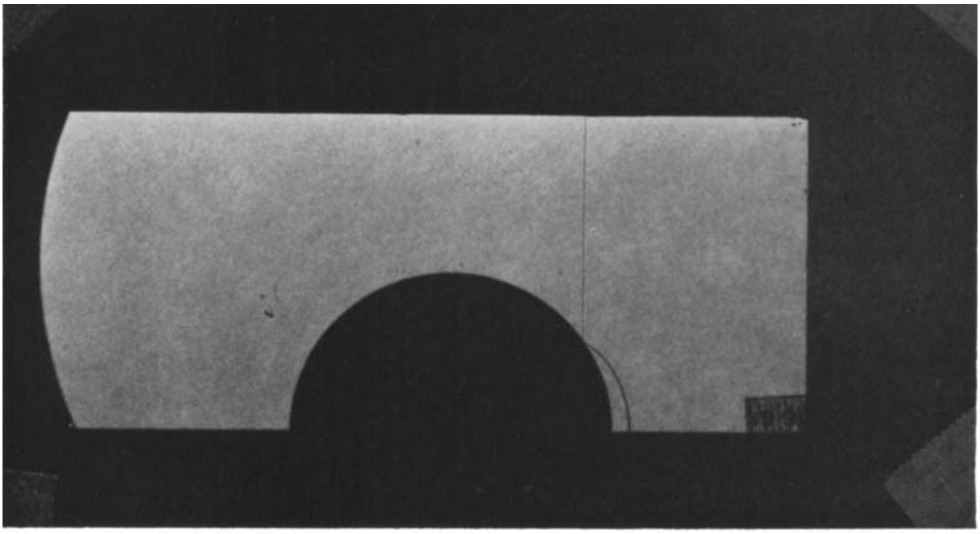


(*a*)



(*b*)

FIGURE 7. Instantaneous shadowgraphs illustrating RR and MR over a concave wedge (model *A*) in dry air. (*a*) Mach reflexion (MR), $M_s = 1{\cdot}40$; (*b*) Regular reflexion (RR), $M_s = 1{\cdot}40$.



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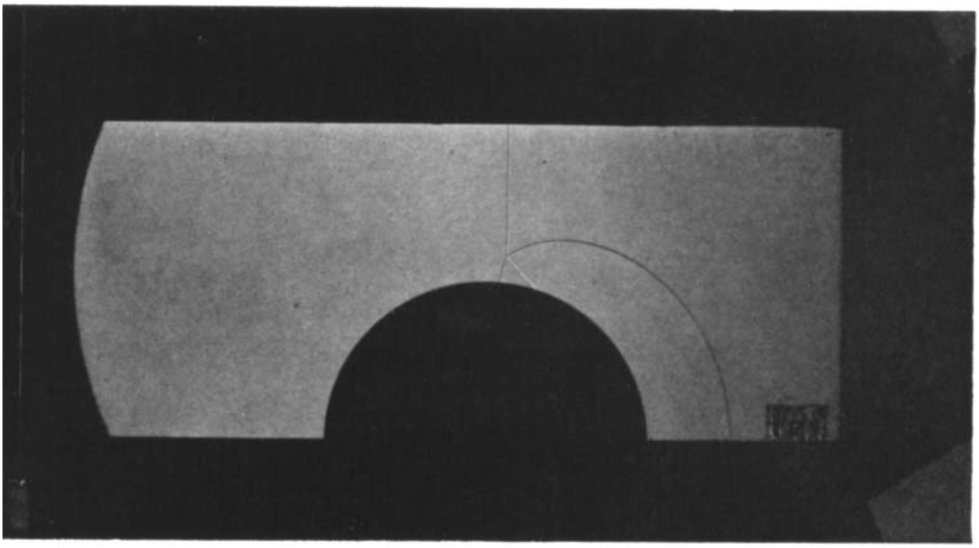
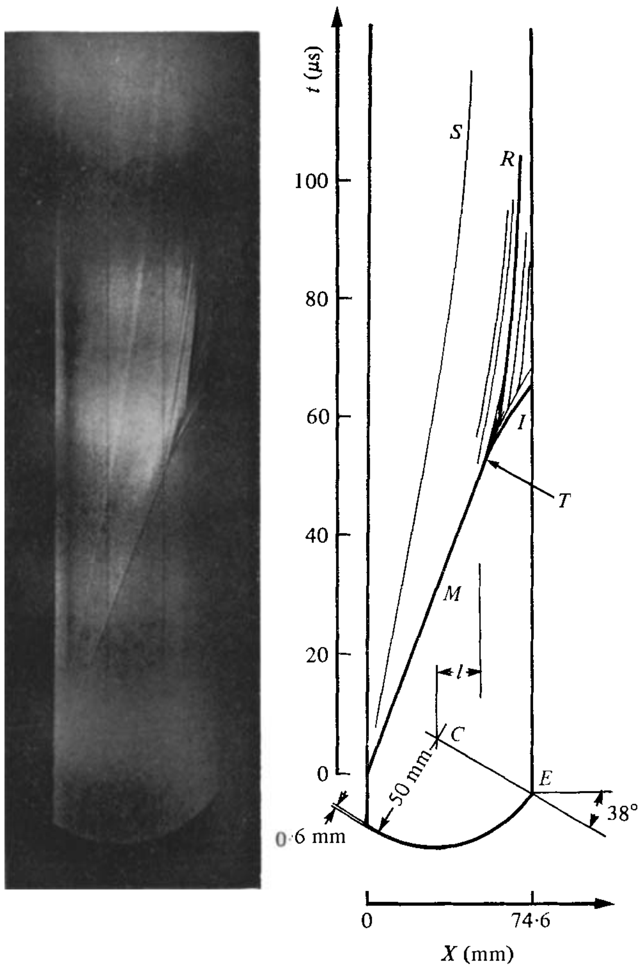


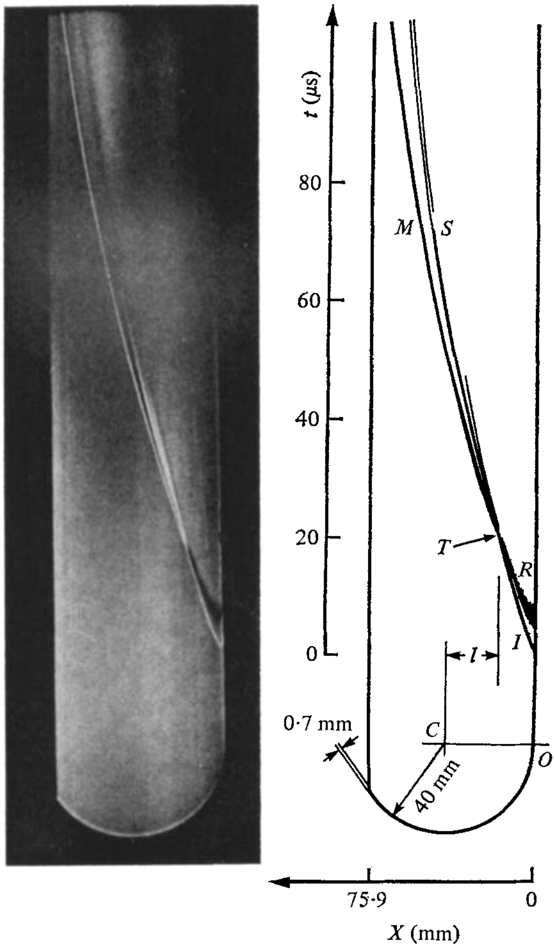
(*b*)

FIGURE 8. Instantaneous shadowgraphs illustrating RR and MR over a convex wedge (model *B*) in dry air. (*a*) Regular reflexion (RR), $M_s = 1{\cdot}40$. (*b*) Mach reflexion (MR), $M_s = 1{\cdot}40$.




(*a*)

FIGURE 9. Streak photographs (on the left) and explanatory sketches (on the right) illustrating actual transitions. *I*, incident shock wave in RR; *R*, reflected shock wave in RR; *M*, Mach stem in MR; *S*, slipstream in MR; *T*, transition point; *C*, centre of the cylindrical surface; *R*, radius of curvature; *l*, horizontal distance between *C* and *T*; $\theta_{w\mathrm{tr}}$, transition wedge angle. (*a*) *MR* → RR (model *A*), $M_s = 2{\cdot}19$; (*b*) RR → MR (model *B*), $M_s = 2{\cdot}80$.




(*b*)

FIGURE 9(*b*). For legend see plate 4.